

Transform Ranking: a New Method of Fitness Scaling in Genetic Algorithms

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Abstract The first systematic evaluation of the effects of six existing forms of fitness scaling in genetic algorithms is presented alongside a new method called transform ranking. Each method has been applied to stochastic universal sampling (SUS) over a fixed number of generations. The test functions chosen were the two-dimensional Schwefel and Griewank functions. The quality of the solution was improved by applying sigma scaling, linear rank scaling, nonlinear rank scaling, probabilistic nonlinear rank scaling, and transform ranking. However, this benefit was always at a computational cost. Generic linear scaling and Boltzmann scaling were each of benefit in one fitness landscape but not the other. A new fitness scaling function, transform ranking, progresses from linear to nonlinear rank scaling during the evolution process according to a transform schedule. This new form of fitness scaling was found to be one of the two methods offering the greatest improvements in the quality of search. It provided the best improvement in the quality of search for the Griewank function, and was second only to probabilistic nonlinear rank scaling for the Schwefel function. Tournament selection, by comparison, was always the computationally cheapest option but did not necessarily find the best solutions.

1 Introduction

Two common forms of selection for reproduction in a genetic algorithm are roulette wheel sampling with replacement and stochastic universal sampling (SUS). Both are forms of fitness-proportional selection, i.e., the probability of an individual being chosen for reproduction is proportional to its fitness. Such approaches are susceptible to both premature convergence and stalled evolution.

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To overcome these problems, fitness scaling methods have been devised to transform the raw fitness, i.e. the objective function, into a scaled selective function used in selecting individuals for reproduction [1]. This paper presents the first systematic analysis and comparison of the performance of a range of six existing fitness scaling methods against two challenging benchmark optimization problems. A new scaling technique called transform ranking is also introduced and evaluated. These seven techniques are also compared with tournament selection, for which the application of fitness scaling would have no effect, since tournament selection is determined by the rank ordering of fitness rather than absolute values.

2 Fitness scaling

Fitness scaling can be applied at the early stages of evolution to weaken selection and thereby encourage exploration of the whole search space. Conversely, at the late stages of evolution, fitness scaling is intended to strengthen the selection pressure in order to converge on the exact optimum. Six existing approaches to fitness scaling are considered here. More detail is available in [1].

Generic linear scaling:

This is a simple linear relationship between the scaled fitness, s_i , and raw fitness f_i . Kreinovich *et al* [2] have demonstrated mathematically that linear scaling is the optimal form of scaling, but only if optimal scaling parameters are known.

Sigma scaling:

Sigma scaling is a variant of linear scaling where an individual's fitness is scaled according to its deviation from the mean fitness of the population, measured in standard deviations (i.e., 'sigma', σ).

Boltzmann scaling:

Boltzmann scaling is a nonlinear method that uses the idea of a "temperature", T , that drops slowly from generation to generation.

Linear rank scaling:

In linear rank scaling, the scaled fitnesses are evenly spread based on the rank ordering of the chromosomes from the fittest to the least fit.

Nonlinear rank scaling:

This is a nonlinear form of rank scaling that increases the selection pressure.

Probabilistic nonlinear rank scaling:

Nolle *et al* [3] have integrated nonlinear rank scaling into roulette wheel selection and SUS, rather than treating it as a separate initial stage.

3 A new scaling algorithm: transform ranking

Linear rank scaling ensures an even spread of scaled fitnesses and hence a lower selection pressure than the nonlinear form. It is therefore suggested that linear rank scaling is well-suited to the early stages of evolution, when exploration of the search space is to be encouraged. It is further suggested that nonlinear rank selection is better suited to the later stages of evolution, when exploitation of the optimum is to be encouraged.

This paper therefore proposes a new form of rank scaling, transform ranking, that progresses from almost linear to increasingly nonlinear. Its basis is probabilistic nonlinear rank scaling:

$$n_i = \text{roundup} \left(\frac{N - Ne^{-cx_i}}{1 - e^{-c}} \right) \quad (1)$$

where n_i is the reverse linear rank of individual i selected by this process for mating, N is the population size, x_i is a set of N random numbers in the range 0–1 (evenly distributed in the case of SUS), c is a constant that controls the degree of nonlinearity, and *roundup* is a function that returns the smallest integer that is not less than its argument.

Nolle *et al* [3] have already shown that Equation 1 is close to linear rank scaling at $c = 0.2$, but becomes highly nonlinear at $c = 3.0$. So the transition between the two modes can be achieved by a progressive increase in c , analogous to the cooling schedule in Boltzmann scaling. The transition schedule can be either linear or geometric:

$$c_{t+1} = c_t + \Delta \quad \text{or} \quad c_{t+1} = \frac{c_t(100 + k)}{100} \quad (2)$$

where c_t and c_{t+1} are the values of c at successive generations, Δ is the increment added at each generation, and k is a percentage increase at each generation.

4 Experimental method

The two-dimensional Schwefel [4] and Griewank [5] functions were used as fitness landscapes for testing the genetic algorithms. Both are symmetric, separable, continuous and multimodal functions. Each reported result is the highest fitness obtained after 50 generations, which was the termination criterion, averaged over 5000 test runs. Initial experiments were carried out to find optimal parameters, which were then retained for all the scaling experiments. Tournament selection was included in the evaluation for comparison purposes only.

5 Results and Discussion

The comparative results of the selection strategies are shown in Fig. 1. For both test functions, the highest fitness solution has been improved through each of the following scaling methods: sigma scaling, linear rank scaling, nonlinear rank scaling, probabilistic nonlinear rank scaling, and transform ranking. Generic linear scaling and Boltzmann scaling were each of benefit for one fitness landscape but not the other.

The best improvement of all was achieved by probabilistic nonlinear rank scaling for the Schwefel function (Fig. 1(a)) and by transform ranking with a linear transform schedule ($\Delta = 0.1$) for the Griewank function (Fig. 1(b)). The success of transform ranking as a new approach to fitness scaling supports the

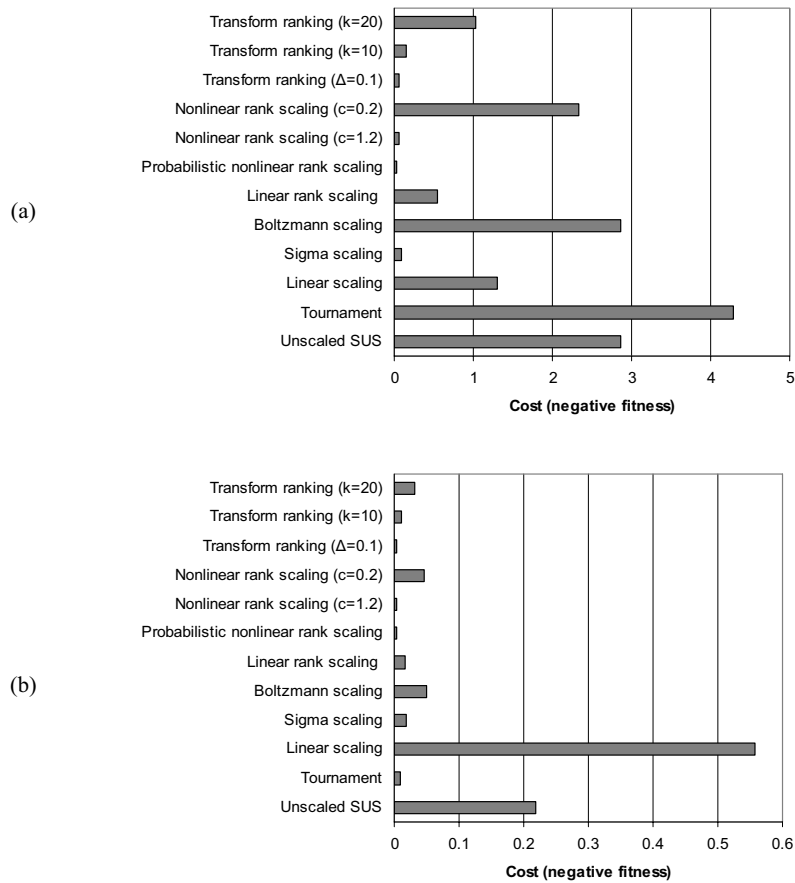


Fig. 1 Best fitness solution using scaled SUS: (a) 2-D Schwefel function, (b) 2-D Griewank function. Tournament selection is included for comparison.

original hypothesis that the transformation from linear to nonlinear rank scaling can lead to improved control of the selection pressure. The improvement is greatest for the linear transform schedule. The geometric transform schedule is highly sensitive to parameter k .

The results show the best fitness obtained, averaged over 5000 test runs. This value was more strongly influenced by the number of times the algorithm failed to reach the global optimum than how effectively the global optimum was exploited. The poor performance of Boltzmann scaling is consistent with the concern expressed by Sadjadi [6] that the method might be susceptible to premature convergence at a local optimum if faced with a complex fitness landscape.

The benefits of fitness scaling always bring a computational cost. Fig. 2 shows the computational costs, normalized with respect to unscaled SUS so that they are machine-independent. The average times for the unscaled SUS were 295s and 299s respectively for the Schwefel and Griewank functions on a 1.5 GHz Inter Pentium computer with 1 GB RAM. The most computationally expensive

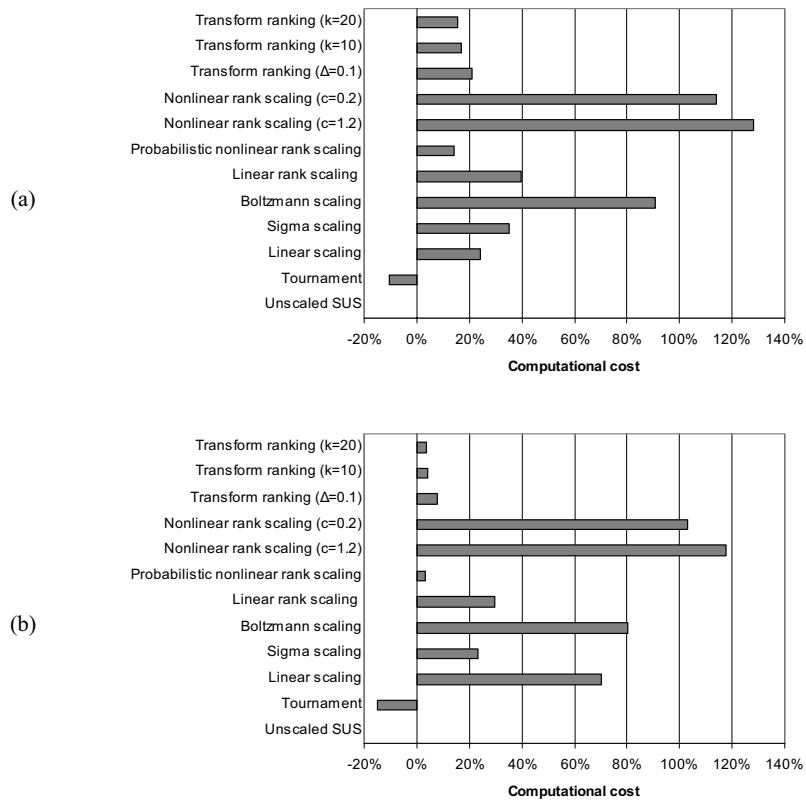


Fig. 2 Computational cost of scaled SUS compared with the unscaled version: (a) 2-D Schwefel function, (b) 2-D Griewank function. Tournament selection is included for comparison.

methods are nonlinear rank and Boltzmann. Encouragingly, the two most effective scaling mechanisms, probabilistic nonlinear rank and transform ranking, are both comparatively inexpensive.

Tournament selection gave poor results for the Schwefel function, but performed much better for the Griewank function. As tournament selection was the computationally cheapest option, it might have found better solutions if the problem had been time bounded rather than bounded by the number of iterations.

6 Conclusions

The benefits of fitness scaling have been demonstrated in searching for the optimum of the two-dimensional Schwefel and Griewank functions. The highest fitness found has been improved through sigma scaling, linear rank scaling, nonlinear rank scaling, probabilistic nonlinear rank scaling, and transform ranking. However, this benefit was always at a computational cost. Although tournament selection performed relatively poorly, particularly against the Schwefel function, it is nevertheless the computationally cheapest option and would therefore have the benefit of additional iterations in time-bounded trials.

A new fitness scaling function, transform ranking, progresses from linear to nonlinear rank scaling during the evolution process, in accordance with a transform schedule. The version with a linear transform schedule provided the best improvement in the quality of search for the Griewank function, and was second only to probabilistic nonlinear rank scaling for the Schwefel function.

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